

Vacuum Expectation Values from a variational approach

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ABSTRACT: In this letter we propose to use an extension of the variational approach known as Truncated Conformal Space to compute numerically the Vacuum Expectation Values of the operators of a conformal field theory perturbed by a relevant operator. As an example we estimate the VEV's of all (UV regular) primary operators of the Ising model and of some of the Tricritical Ising Model when perturbed by any choice of relevant primary operators. We compare our results with some other independent predictions.

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1 Introduction: the need of a variational approach for VEVs

One physical application of ($D = 2$) conformal field theories, [1], is that they can be successfully used to describe the critical properties of statistical systems, such as critical indices and critical correlators, (see e.g. [2]). Furthermore, following this line of thought, the behavior of the system near criticality, can be obtained by analyzing a conformal field theory perturbed by one or more relevant operators i.e. when to the conformal action S_{CFT} is added a perturbation of the form

$$\Delta S = \int dx \sum_i \lambda_B^i \Phi_{iB}(x), \quad (1.1)$$

where λ_B^i (Φ_{iB}) are bare couplings (operators) and $0 < x_i < 2$ ($x_i \equiv x_{\Phi_i}$, x_O being the scale dimension of operator O).

A wide class of perturbations have been found to generate integrable models, [3], for which the S matrix is known exactly and for which the off shell behavior of the theory can be estimated by use of the form factor method [4], that gives rise to a long distance expansion for correlators.

Another approach to the problem [5, 6] is obtained by using perturbative expansions around the (massless) conformal field theory, from which one could get informations on the short distance behavior of the complete theory. This point of view is important not only because it is complementary to the previous one, but also because it is in principle independent on the integrability of the model. The main problem in this case is the presence of I.R. divergences that arise in the naive Gell Mann Low series when the perturbing operator is relevant. These IR problems can be solved by use of the Operator Product Expansion (OPE). The idea of this approach came out very early in the general context of quantum field theories and statistical mechanics[7] (see [8] for similar ideas in QCD), was first introduced in the context of perturbed conformal theories by Al. Zamolodchikov in [9] (see also [10] for an analog independent proposal), and was developed at all orders in a very general context in [12].

The idea of the OPE approach is the fact that the so-called Wilson coefficients $C_{ab}^c(r, \lambda)$ that enter in the Operator Product Expansion for the complete theory,

$$\langle \Phi_a(r) \Phi_b(0) \rangle_\lambda \sim \sum_c C_{ab}^c(r, \lambda) \langle \Phi_c(0) \rangle_\lambda \quad (1.2)$$

are essentially short distance objects that can be taken to have a regular, I.R. safe, perturbative expansion with respect to the renormalized couplings

λ^i , while non analytic contributions to correlator can be confined in the (non perturbative) Vacuum Expectations Values (VEV's). Main result of [12] is an I.R. finite representation for the n^{th} derivative of C_{ab}^c with respect to the couplings evaluated at $\lambda^i = 0$, involving integrals of (eventually renormalized) conformal correlators. The first order formulas have been applied to two models very interesting from the point of statistical mechanics and integrable systems: the Ising model (IM) in external magnetic field [13] and the Tricritical Ising Model (TIM) at $T > T_c$, [14]. (See also [20] and [11] for earlier applications.) The results obtained were found to be in agreement with form factor estimates [15, 16, 17] in the intermediate region.

It should be evident to the reader that the perturbative knowledge of Wilson coefficients is not sufficient to fully reconstruct correlators: actually one has to give some informations on VEV's.

The first, main information on VEV's is obtained from the Renormalization Group equations for the operators to which we are interested: as well known in perturbative quantum field theories and from direct RG considerations, see e.g. [10], renormalized ¹ composite operators VEV's evolve by RG as

$$\frac{d}{dl}\langle[\Phi_a]\rangle = \Gamma_a^b\langle[\Phi_b]\rangle \quad (1.3)$$

in which l is the logarithm of the scale and the matrix Γ_a^b contains only positive integer powers of the renormalized couplings λ^i and forbids the mixing with higher dimensional operators (it is lower diagonal if operators are ordered with increasing dimensions). It is clear from Eq.(1.3) that when one coupling is considered, (as is the case for this paper) the mixing of two operators, responsible of the presence of logarithms of λ , could happen only if the difference of two operators is equal to a positive multiple of the dimension of λ (resonance condition). When this is not the case the only nonperturbative information on operators VEV's is contained in a constant, noted by A below:

$$\langle[\Phi_{\Delta'}]\rangle = A(\Delta', \Delta)|\lambda|^{\frac{\Delta'}{1-\Delta}} \quad (1.4)$$

(Δ being the scale dimension of the perturbing operator). In the following, to avoid UV problems we will limit ourselves to compute the VEV's of operators that do not need any multiplicative renormalization² and for which the resonance condition does not hold.

If the complete theory is integrable, the knowledge of the S matrix gives some informations on the bulk energy that can be recovered from the expression of the free energy in terms of the mass scale as obtained from the

¹We denote by $[\Phi]$ any renormalized operator Φ

² This condition is satisfied for an operator $\Phi_{\Delta'}$ if $\Delta + \Delta' < 1$, as easily seen from an (IR regulated) perturbative expansion analysis for the VEV in question.

Thermodynamic Bethe Ansatz approach, [21, 22]. In this case (if the relation between the main mass and λ is known, [23, 22]) the VEV of the perturbing operator is simply the derivative with respect to λ of the free energy. These informations are available for all the $\mathcal{M}(\xi) + \phi_{1,3}$ models as well as all the $\mathcal{M}(\xi) + \phi_{1,2}$ ($\mathcal{M}(\xi) + \phi_{2,1}$).³ In particular they were used in the papers [20, 11, 12, 13].

Moreover very recently Lukyanov and A. Zamolodchikov[24] gave an exact prediction for the VEV's of the type $\langle e^{ia\phi} \rangle$ in the Sine-Gordon model that, by quantum group reduction, was translated in the knowledge of all VEV's of primary operators of all $\mathcal{M}(\xi) + \phi_{1,3}$ theories.

In this letter we propose to extend the use of a variational approach, known as Truncated Conformal Space approach, first introduced by Al. Zamolodchikov in the context of perturbed conformal field theories [20], to compute numerically the constants A above. The interest in our extension is mainly that it completes the framework of short distance behavior of perturbed conformal field theories *without* requiring the integrability of the model. Moreover the estimates for TIM and IM that we give here as an example, constitute an independent check of previous evaluation of constants A and in particular an independent check of the Lukyanov Zamolodchikov expressions.

In next section we review the variational method and the technical observations of the present case. In section 3 we describe the numerical analysis and the results for the two examples comparing with known results, while conclusions are left to section 4.

2 The method

To deal with the nonperturbative IR information, we need an IR regularization of the complete theory on the whole plane: to do that we will use the corresponding field theory on an infinite cylinder of circumference R and we will recover the original theory by taking the limit $R \rightarrow \infty$. The regularized theory will be seen as a conformal theory perturbed by an operator $\tilde{\Phi}_\Delta$ (the tilde will refer to quantities of the regularized theory). The present approach assumes implicitly that the operators of the complete theories on the cylinder and on the plane are labeled by the operators of the unperturbed conformal theory, and that in the limit $R \rightarrow \infty$ states and operators on the

³We use the notation $\mathcal{M}(\xi)$, $\Phi_{l,k} \equiv \Phi_{\Delta_{l,k}}$ to denote respectively the minimal conformal theory with central charge $c = 1 - \frac{6}{\xi(\xi+1)}$ and the scalar primary operator of scale dimension $\Delta_{l,k} = \frac{((\xi+1)l - \xi k)^2 - 1}{4\xi(\xi+1)}$ and its extension on the complete theory.

cylinder tend to corresponding quantities of the plane. The hamiltonian of the regularised theory will have the form:

$$\tilde{H} = \tilde{H}_0 + \tilde{V} \quad (2.1)$$

where \tilde{H}_0 is the Hamiltonian of the conformal theory on the cylinder and $\tilde{V} = \int_0^R dw_2 \tilde{\Phi}_\Delta(w)$ is the perturbation ($w = w_1 + iw_2$). By use of the conformal transformation $z = e^{\frac{2\pi}{R}w}$ the conformal theory on the cylinder can be shown to be equivalent to a theory on a punctured plane where z lives (this equivalence explains why in the limit $R \rightarrow \infty$ we recover the theory on the plane). In terms of quantities of the auxiliary punctured plane (denoted by an hat) we can write

$$\tilde{H}_0 = \frac{2\pi}{R}(\hat{L}_0 + \hat{\bar{L}}_0 - c/12) \quad (2.2)$$

$$\tilde{\Phi}_\Delta(w) = \left| \frac{2\pi z}{R} \right|^{2\Delta} \hat{\Phi}_\Delta(z) \quad (2.3)$$

(we will fix without any loss of generality the quantization "radial time" $w_1 = 0$, corresponding to $|z| = 1$).

The key property that we will use is the fact that VEV's of the complete theory on the plane can be recovered as a limit of the corresponding object on the cylinder:

$$\langle 0 | \Phi_{\Delta'} | 0 \rangle = \lim_{R \rightarrow \infty} \langle \tilde{0} | \tilde{\Phi}_{\Delta'} | \tilde{0} \rangle. \quad (2.4)$$

Next step is to use the Truncated Conformal Space approach, to deal numerically with the theory at finite R , as first proposed in [20]. The TCS variational approach consists in truncating the conformal space of states (supposed to be the same for the complete theory) to some space of finite dimension N (obtained by giving an upper limit on values of $x \equiv L_0 + \bar{L}_0$) and to diagonalize the hamiltonian that will be described by a self adjoint matrix in this case.

More in detail, denoting the set of (a priori nonorthogonal) conformal states with $\{|i\rangle\}_1^N$, one must diagonalize the $N \times N$ hamiltonian:

$$H_j^i \equiv g^{ik} \langle k | \tilde{H} | j \rangle \quad (2.5)$$

(g^{ik} above is the inverse of the matrix $g_{ik} = \langle i | k \rangle$ and is added to keep into account the nonorthogonality of the chosen basis that manifests also in the modified resolution of the identity $\mathbf{1} = |i\rangle g^{ij} \langle j|$). Remembering Eqs.(2.1-2.3) it follows:

$$H_j^i = \frac{2\pi}{R} \left[(x_i - c/12) \delta_{ij} + 2\pi \lambda \left(\frac{R}{2\pi} \right)^{2(1-\Delta)} g^{ik} \langle k | \hat{\Phi}_\Delta | j \rangle \right]. \quad (2.6)$$

The matrix elements $\langle k|\hat{\Phi}_\Delta|j\rangle$ can be reconstructed from the knowledge of Wilson coefficients of the conformal theory on the punctured plane, see [20].

TCS method has been used successfully to recover information on the spectrum of the complete theory on the cylinder by use of the secular equations for H_j^i :

$$\det [H_j^i - E^{(N)}\delta_j^i] = 0. \quad (2.7)$$

It is clear that this method should converge for fixed R when N is big enough. When this condition is no more satisfied unphysical truncations errors are important and results are no more reliable. As suggested in [20] a check on the behavior $E_0^N \sim R$ (when this is the case) in the $R \rightarrow \infty$ region by numerically verifying if

$$\frac{d \log E_0^{(N)}}{d \log R} = 1 \quad (2.8)$$

is a good control on the choices of R and N .

The step forward that we propose here is to compute also eigenvectors $\psi^{(N)}$ of H_j^i , in particular the ground state vector $\psi_0^{(N)}$: from this information the VEV's of the complete theory can be obtained by taking numerically the double limit $R \rightarrow \infty, N \rightarrow \infty$:

$$A(\Delta', \Delta) \equiv \langle \Phi_{\Delta'} \rangle / |\lambda|^{\frac{\Delta'}{1-\Delta}} = \lim_{R, N \rightarrow \infty} \langle \tilde{0} | \tilde{\Phi}_{\Delta'} | \tilde{0} \rangle_N / |\lambda|^{\frac{\Delta'}{1-\Delta}} \quad (2.9)$$

$$\langle \tilde{0} | \tilde{\Phi}_{\Delta'} | \tilde{0} \rangle_N \equiv \left(\frac{2\pi}{R} \right)^{2\Delta'} \langle \tilde{0} | \hat{\Phi}_{\Delta'} | \tilde{0} \rangle_N \quad (2.10)$$

$$\langle \tilde{0} | \hat{\Phi}_{\Delta'} | \tilde{0} \rangle_N \equiv \left(\psi_0^{(N)i} \langle i | \hat{\Phi}_{\Delta'} | j \rangle \psi_0^{(N)j} \right) / \left(\psi_0^{(N)i} g_{ij} \psi_0^{(N)j} \right). \quad (2.11)$$

Another useful test on R, N (more sensitive to the VEV one is considering) is to verify if $\langle \tilde{0} | \tilde{\Phi}_{\Delta'} | \tilde{0} \rangle_N \sim R^{2\Delta'}$ (as imposed by the fact that $A(\Delta', \Delta)$ is a finite number), i.e. to check numerically if

$$\frac{1}{(2\Delta')} \frac{d \log \langle \tilde{0} | \tilde{\Phi}_{\Delta'} | \tilde{0} \rangle_N}{d \log R} = 1. \quad (2.12)$$

3 Examples: IM and TIM

In this section we will apply the ideas delined in the previous section to the case of the Ising Model and Tricritical Ising Model (respectively $\mathcal{M}(3)$ and $\mathcal{M}(4)$ conformal theories). The primary scalar operators of the theory are (in the notation (Φ_Δ, Δ)):

$$IM = \{(1, 0), (\sigma, 1/16), (\varepsilon, 1/2)\}$$

$\pm\Delta, \text{ vac.}$	1/16	1/2
$+1/16, \text{ vac}$	$-1.277(2)$	$1.94(6)$
$+1/2, \text{ sym}$	0	<i>U.V.</i>
$-1/2, \text{ ssb}$	$\pm 1.69(2)$	<i>U.V.</i>

Table 1: Coefficients $A_{\Delta, \Delta'}$ for perturbed Ising Model (see Eq.(5)): Δ' is reported at the top of each column; the sign before Δ refers to the sign of the coupling; see text for explanations about the different vacua.

$$TIM = \{(1, 0), (\sigma, 3/80), (\varepsilon, 1/10), (\sigma', 7/16), (\varepsilon', 3/5), (\varepsilon'', 3/2)\}$$

The perturbation we will consider are σ, ε for IM and $\sigma, \varepsilon, \sigma', \varepsilon'$ for TIM.

For the numerical elaboration we used a slight modification of the Mathematica program STRIP [19] that can deal minimal conformal field theories up to level 5 (i.e. up to $N = 43$ states for IM and $N = 228$ for TIM). The result of the analysis are reported in Table (1) for IM and in Table (2) for TIM. According to the type of the perturbations the spin reversal symmetry of the conformal theory can be exact, spontaneously broken or explicitly broken in the complete model and we denoted the corresponding vacuum state (of the theory on the cylinder) respectively by "sym, ssb, vac". A detailed analysis of the structure of vacua in TIM can be found in [18]. The values 0 in tables are fixed by spin reversal symmetry, while with "U.V." we mean that the resonance condition is satisfied and that the VEV is no more of the form (1.4).

The numerical strategy we used was (for a fixed value of N) to look for the values of R such that Eq.(2.12) is satisfied and to have a look at what happens at Eq.(2.8). The (indicative) errors reported are fixed by difference with the result at $N = 8$ for IS, $N = 22$ for TIM. The optimal situation (that we found in many cases) is that when Eq.(2.12) is satisfied Eq.(2.8) holds within $1 - .1\%$.

The general tendency we observe is that truncation errors are bigger for a perturbation Φ_{Δ} of higher Δ , and for fixed perturbation are bigger for VEV's $\langle \Phi_{\Delta'} \rangle$ of higher Δ .

More difficult (due to truncation errors) are the cases $TIM + \sigma'$ and $TIM - \varepsilon'$ ⁴ for which the truncation errors are bigger and stability is not achieved at the values of N considered.

⁴In $TIM + \varepsilon'$ all the VEV's of relevant operators are 0 by symmetry apart from $\langle \varepsilon' \rangle$ that contains log's.

$\pm\Delta, \text{ vac.}$	3/80	1/10	7/16	3/5
$+\frac{3}{80}, \text{ vac}$	-1.5396(8)	1.336(3)	-1.55(3)	1.92(6)
$+\frac{1}{10}, \text{ sym}$	0	-1.466(7)	0	3.4(2)
$-\frac{1}{10}, \text{ ssb}$	$\pm 1.594(2)$	1.466(4)	$\pm 2.38(6)$	3.5(2)

Table 2: Coefficients $A_{\Delta, \Delta'}$ for perturbed Tricritical Ising Model (see Eq.(5)): Δ' is reported at the top of each column; the sign before Δ refers to the sign of the coupling; see text for explanations about the different vacua.

In $TIM + \sigma'$ case⁵ we can estimate (unsymmetric vacuum):

$$A(3/80, 7/16, +, 4) \sim -1.1 \quad (3.1)$$

$$A(1/10, 7/16, +, 4) \sim 1.2 \quad (3.2)$$

while $\langle \varepsilon' \rangle$ contain log's, and $\langle \sigma' \rangle$ is very unstable (this it not a surprise since it was observed [28] that the level of accuracy of the STRIP program is not enough to describe the ground energy of which $\langle \sigma' \rangle$ is essentially the derivative)

In the $TIM - \varepsilon'$ case we obtained

$$A(3/80, 3/5, -, 4) \sim \pm 1.4 \quad (3.3)$$

while $\langle \varepsilon \rangle$ is not yet stabilized.

We report now some predictions on the VEV's that can be found in the literature (see footnote 5 for convention used).

First of all we quote the exact predictions on the VEV of the perturbation coming from the Thermodynamic Bethe Ansatz [21, 22, 23]

$$A(1/16, 1/16, +, 3) = -1.27758227605119295 \dots \quad (3.4)$$

$$A(1/10, 1/10, +, 4) = -1.468395424027621489 \dots \quad (3.5)$$

Secondly from the general formulas of Lukyanov and Zamolodchikov [24] we can derive:

$$A(1/16, 1/2, -, 3) = 1.7085219053746979652 \dots \quad (3.6)$$

$$A(3/80, 3/5, -, 4) = 1.4105216239133927375 \dots \quad (3.7)$$

⁵ We will use in the following the notations

$$\langle \Phi_{\Delta'} \rangle = A(\Delta', \Delta, \mathbf{Sign} \lambda, \xi) |\lambda|^{\frac{\Delta'}{1-\Delta}}$$

for the theory $\mathcal{M}(\xi) + \lambda \Phi_{\Delta}$.

We report then some results obtained by the authors in the previous analysis of critical Ising Model in magnetic field (from fit of lattice data and the imposition of C and Δ exact sum rules, [25, 26])

$$A(1/2, 1/16, +, 3) = 2.02(10) \quad (3.8)$$

(we reported an error of 5% as suggested in the paper) as well as in TIM plus ϵ (C and Δ exact sum rules imposed)

$$A(3/5, 1/10, +, 4) = 3.8(4). \quad (3.9)$$

The estimate

$$A(1/2, 1/16, +, 3) \sim 1.95 \quad (3.10)$$

has been obtained recently by use of MonteCarlo simulations, [27].

It appears that all the predictions are consistent with our results within the given errors!

4 Conclusions

The purpose of this letter was to suggest that a variational method could be efficient to recover (numerical) non perturbative informations on VEV's *independently* of integrability of the underlying perturbed conformal theory. We want to emphasize in particular that such a method is in principle capable of giving informations on all VEV's of the UV regular scalar operators of the complete theory, secondary operators and multiple couplings perturbations included.

As an example of the application of the method we considered the perturbed IM and TIM, and we studied the VEV's of primary operators of those models with a slight modification of an existing program [19], obtaining encouraging and nontrivial results already at level 5. Present results are in agreement within errors with exact results from TBA [21, 23] and from Lukyanov and A. Zamolodchikov [24], with some previous existing estimates of the authors, [13, 14] and with MonteCarlo numerical analysis [27]. Moreover new original estimates have been given for some VEV's, see tables.

High precision estimates of constants A , parameterizing UV regular VEV's, are realizable with a more sophisticated (but not un-feasible) program to deal with conformal states and matrix elements, as explained in [20, 18, 19] and are outside the goal of this paper.

We conclude by observing that the results of this letter should be considered as an important step forward towards the practical description of short distance behavior of statistical mechanical systems in two dimension by use of the OPE approach: the all order formulas for the Wilson coefficients developed in [12] together with the present approach constitute a complete, consistent tool that can reach any accuracy, at least in the case of absence of UV divergences.

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